

Dirac Delta : <sup>Q1:</sup> how to generate this continuous-time signal in the Lab?  
 ⇒ we can't build a real Dirac delta (infinite / undefined amplitude)  
 ⇒ how to create something LIKE a Dirac Delta?

(Lecture slide 3-5) <sup>A1:</sup> but we can build a very thin square wave!  
 the question is: how thin? (<sup>Q2:</sup> how short should the square wave last?)

<sup>A2:</sup> ⇒ depends on application (how short it needs to be)  
 and physical constraints (how short it can be)

<sup>Q3:</sup> what about the height/amplitude of the square wave?

<sup>A3:</sup> ⇒ usually on order of 10's of Volts  
 (large enough amplitude)

but amplitude of square wave is of less concern  
 b/c it can easily be adjusted in DSP

ECE 445S  
 Real-Time DSP Lab

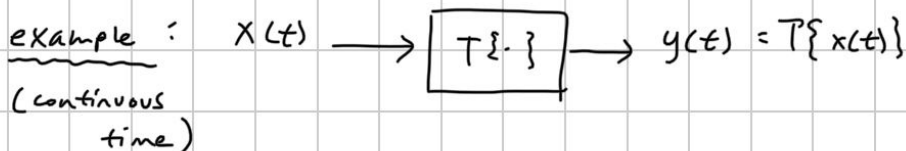
Lecture 3 on  
 Signals and Systems

January 30, 2022  
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⋮

Systems (slide 3-7)

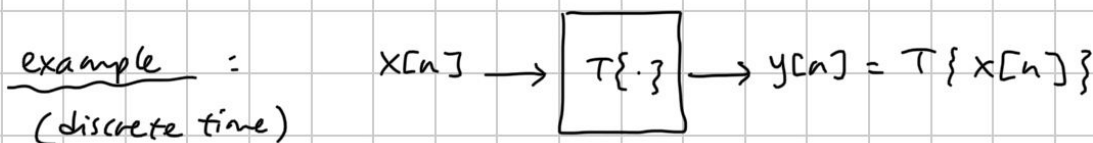
→ can also operate on MULTIPLE inputs  
 but, we start off with 1 input, 1 output system



$$y(t) = \frac{x(t) + x(t-1)}{2}$$

output = averaging fn.  
 averages current & previous <sup>input</sup> sample from 1 second ago  
 this is kind of a large time gap!

or  $y(t) = x^2(t)$  (Squaring block)



$$y[n] = \frac{x[n] + x[n-1]}{2}$$

★ this is an averaging Filter  
 also can be a Low Pass Filter

## Initial Conditions for Linear Systems

we only get to observe signals & systems starting at time  $t=0$

example : Integrator (C.T. system)

$$y(t) = \int_0^t x(u) du + C_0$$

↑ initial condition.

does  $y(t)$  satisfy linearity?

check additivity:  $x_1(t) + x_2(t) \rightarrow \boxed{\int(\cdot)}$   $\rightarrow y_1(t) + y_2(t)$ ?

↓  
satisfied  
regardless of  $C_0$

$$\Rightarrow \underbrace{\left( \int_0^t x_1(u) du + C_0 \right)}_{y_1(t)} + \underbrace{\left( \int_0^t x_2(u) du + C_0 \right)}_{y_2(t)}$$

what about homogeneity?  $a x(t) \rightarrow \boxed{\int(\cdot)}$   $\rightarrow a y(t)$ ?

$$\Rightarrow a x(t) \stackrel{?}{=} a (y(t))$$

$$\int_0^t (a x(u)) du + C_0 \stackrel{?}{=} a \left( \int_0^t x(u) du + C_0 \right)$$

$$a \int_0^t x(u) du + C_0 \stackrel{?}{=} a \int_0^t x(u) du + a \cdot C_0$$

\* homogeneity holds for all constant scalar  $a$   
if  $C_0 = 0$ , otherwise

homogeneity may not hold!!

$\Rightarrow$  then system is NOT linear!!

$\Rightarrow$  Initial conditions must be zero as a necessary condition for the system to be linear

Initial Conditions for Time Invariant Systems?  $\Rightarrow$  refer to Handouts on slide 3-13

(break at 11:16 PM)

(return @ 11:22 PM for In-Lecture Assignment)

In-Lecture Assignment, (also HW 1,2)

# Continuous Time (C.T.) System Properties

continuous time  
"LTI"  
system

⇒ Linearity :

- ① additivity
- ② homogeneity

$$\begin{aligned} [x_1(t) + x_2(t) &\Rightarrow y_1(t) + y_2(t)] \\ [a \cdot x_1(t) &\Rightarrow a \cdot y_1(t)] \end{aligned}$$

these are  
2 independent  
tests!!

Q4 how to quickly tell that a system does not satisfy linearity?

A4 ⇒

Use that homogeneity has to hold for any constant  $a$ . Let  $a = 0$ . This means that for an input signal that is zero for all time, and the output signal would be zero for all time for a homogeneous system. If a system fails this quick test, then it is nonlinear; otherwise, we have prove that the properties hold.

⇒ Time Invariance

Why do we like LTI system? (slide 3-9)

- ⇒ can completely characterize system with impulse response (dirac delta)
- ⇒ can hide detailed system & implementation inside large black box



Examples on testing Linearity (additivity, homogeneity) and Time Invariance of Systems ⇒ (slide 3-10, 11)

## Chirp Signal (slide 1-14)

continuous time sinusoidal signal

has rising/falling frequency vs. time

has non-zero bandwidth

⇒ finite observation time (mult. by rect or square wave)

⇒ can refer back to HW 0.1

→ aka. "linear sweeps"

rising frequency = up sweep (?)

falling frequency = down sweep

implementing a chirp depends on... 3 parameters ( $A, \mu, f_0$ )

$$\text{chirp } x(t) = A \cdot \cos(2\pi\mu t^2 + 2\pi f_0 t)$$

for  $t$  ranging:  $0 \leq t \leq t_{\max}$ ,

for a chirp from  $f_1$  to  $f_2$ ,

the parameters should be ...  $f_0 = f_1$

$$\mu = (f_2 - f_1) / (2t_{\max})$$

$A$  = amplitude of chirp signal, this has to do with volume of chirp.

this can be set arbitrarily (as long as chirp is audible)

## ★ Introduction to Spectrogram

time vs. frequency vs. amplitude (like a 3D plot)

we can view the signal in time & frequency domains

frequency resolution =  $f_s / N$

time resolution = shift =  $N$ -overlap

(MATLAB demo @ 11:43 AM)

(many chirp sounds @ 11:47 AM :))

## additional notes on Spectrogram Resolutions ...

to improving frequency resolution  $\Rightarrow$  this is related to  $f_s/N$

$f_s$  = sampling freq.

$N$  = # samples in 1 segment

$\rightarrow$  this means we want to make freq. resolution finer.

$\Rightarrow$  we want  $f_s/N$  to decrease.

this can be done by increasing  $N$

(and vice versa to decrease frequency resolution = make resolution coarser)

to improving time resolution  $\Rightarrow$  this is related to  $N T_s$

In a particular block of  $N$  samples,  
we do not know when specific  
frequency components occur.

$\downarrow$   
increasing time resolution means decreasing  $N$