ECE 445S
Real-Time DSP Lab

Lecture 3 on
Signals and Systems

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Dirac Delta : Q_1: how to generate this continuous-time signal in the Lab?

⇒ we can't build a real Dirac delta (infinite / undefined amplitude)

⇒ how to create something LIKE a Dirac Delta?

(Lecture Slide 3-5) A_1: ↳ but we can build a very thin square wave!

the question is: how thin? (Q_2: how short should the square wave last?)

A_2: ⇒ depends on application (how short it needs to be) and physical constraints (how short it can be)

Q_3: what about the height/amplitude of the square wave?

A_3: ⇒ usually on order of 10's of Volts (large enough amplitude)

but amplitude of square wave is of less concern b/c it can easily be adjusted in DSP

⋮

Systems (slide 3-7)

→ can also operate on MULTIPLE inputs

but, we start off with 1 input, 1 output system

example : (continuous time)

$$x(t) \longrightarrow \boxed{T\{\cdot\}} \longrightarrow y(t) = T\{x(t)\}$$

$$y(t) = \frac{x(t) + x(t-1)}{2}$$

output = averaging fxn.

averages current & previous input sample from 1 second ag

this is kind of a large time gap!

or $y(t) = x^2(t)$ (Squaring block)

example : (discrete time)

$$x[n] \longrightarrow \boxed{T\{\cdot\}} \longrightarrow y[n] = T\{x[n]\}$$

$$y[n] = \frac{x[n] + x[n-1]}{2}$$

☆ this is an averaging Filter

also can be a Low Pass Filter

Initial Conditions for Linear Systems

we only get to observe signals & systems starting at time $t=0$

example : Integrator (C.T. system)

$$y(t) = \int_0^t x(u)du + C_0$$

initial condition.

does $y(t)$ satisfy Linearity?

check additivity : $x_1(t) + x_2(t) \rightarrow \boxed{S(\cdot)} \rightarrow y_1(t) + y_2(t)$?

satisfied regardless of C_0

$$\Rightarrow \underbrace{\left(\int_0^t x_1(u)\,du + C_0\right)}_{y_1(t)} + \underbrace{\left(\int_0^t x_2(u)\,du + C_0\right)}_{y_2(t)}$$

what about homogeneity? $a\,x(t) \longrightarrow \boxed{S(\cdot)} \longrightarrow a\,y(t)$?

$$\Rightarrow a\,x(t) \overset{?}{=} a\,(y(t))$$

$$\int_0^t (a\,x(u))du + C_0 \overset{?}{=} a\left(\int_0^t x(u)du + C_0\right)$$

$$a\int_0^t x(u)du + C_0 \overset{?}{=} a\int_0^t x(u) + a \cdot C_0$$

☆ homogeneity holds for all constant scalar a if $C_0 = 0$. otherwise homogeneity may not hold !!

$\Rightarrow$ then system is NOT linear !!

$\Rightarrow$ Initial conditions must be zero as a necessary condition for the system to be linear

Initial Conditions for Time Invariant Systems?

$\Rightarrow$ refer to Handouts on slide 3-13

(break at 11:16 AM)

(return @ 11:22PM for In-Lecture Assignment)

In-Lecture Assignent, (also HW 1,2)

Continuous Time (C.T.) System Properties

continuous time "LTI" system

⇒ Linearity : ① additivity $[x_1(t) + x_2(t) \Rightarrow y_1(t) + y_2(t)]$

② homogeneity $[a \cdot x_1(t) \Rightarrow a \cdot y_1(t)]$

these are 2 independent tests!!

Q4 how to quickly tell that a system does not satisfy linearity?

A4 ⇒ Use that homogeneity has to hold for any constant a. Let a = 0. This means that for an input signal that is zero for all time, and the output signal would be zero for all time for a homogeneous system. If a system fails this quick test, then it is nonlinear; otherwise, we have prove that the properties hold.

⇒ Time Invariance

Why do we like LTI system? (slide 3-9)

⇒ can completely characterize system with impulse response (dirac delta)

⇒ can hide detailed system & implementation inside large black box

$$\underset{\text{input}}{x(t)} \rightarrow \boxed{\text{System}} \rightarrow \underset{\text{output}}{y(t)}$$

Examples on testing Linearity (additivity, homogeneity) and Time Invariance of Systems ⇒ (slide 3-10, 11)

Chirp Signal (slide 1-14)

continuous time sinusoidal signal
has rising/falling frequency vs. time
has non-zero bandwidth

⇒ finite observation time (mult. by rect or square wave)

⇒ can refer back to HW 0.1

→ a.k.a. "linear sweeps"

rising frequency = up sweep (?)

falling frequency = down sweep

implementing a chirp depends on... 3 parameters (A, μ, f_0)

chirp $x(t) = A \cdot \cos(2\pi \mu t^2 + 2\pi f_0 t)$

for t ranging: $0 \leq t \leq t_{max}$,

for a chirp from f_1 to f_2,

the parameters should be ... $f_0 = f_1$

$\mu = (f_2 - f_1) / (2 t_{max})$

A = amplitude of chirp signal, this has to do with volume of chirp.

this can be set arbitrarily (as long as chirp is audible)

☆ Introduction to Spectrogram

time vs. frequency vs. amplitude (like a 3D plot)

we can view the signal in time & frequency domain

frequency resolution: f_s / N

time resolution : shift = N - overlap

(MATLAB demo @ 11:43 AM)

(many chirp sounds @ 11:47 AM :))

additional notes on Spectrogram Resolutions ...

to improving frequency resolution ⇒ this is related to f_s/N

f_s = sampling freq.

N = # samples in 1 segment

↳ this means we want to make freq. resolution finer.

⇒ we want f_s/N to decrease.

this can be done by increasing N

(and vice versa to decrease frequency resolution = make resolution coarser)

to improving time resolution ⇒ this is related to N Ts

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In a particular block of N samples, we do not know when specific frequency components occur.

increasing time resolution means decreasing N .